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Divisibility Test of Numbers by Some Prime Ones (7, 13, 17, 19, 23, 29)

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Abstract

The purpose of this paper is finding rules to know whether any number is divisible by some primes (7, 13, 17, 19, 23, 29), so that we use divisibility test and concept of congruence relation in numbers theory. The conclusion of the study arrived that we can know that any natural number is divisible by any prime without doing division.

Keywords: *Divisibility Test, Prime, Congruence.*

اختبارقابلية قسمة الأعدادعلى بعض الأعداد الأولية [7, 13, 17, 19, 23, 29]

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مُسْتَخْلَص

الغرض من هذه الورقة هو إيجاد قواعد لمعرفة ما إذا كان أيَّ عدد يقبل القسمة على بعض الأعداد الأولية [7, 13, 17, 19, 23, 29]ولذلك استخدمنا قابلية القسمة ومفهوم التطابق في نظرية العدد. وخلصنا إلى أنه يمكن معرفة قابلية القسمة لأيَّ عدد طبيعي على عدد أولي بدون إجراء عملية القسمة.

كلمات مفتاحية:*اختبارقابلية القسمة، الأعداد الأولية، التطابق.*

Introduction

Number theory contains the concept of congruence which considered other expression of division. Divisibility test of numbers by some prime (7, 13, 17, 19, 23, 29) has rules which used to check whether any positive integer is divided by these prime numbers. Applying the methods adding or subtracting, the case may be from the remaining truncated number is divisible by the given number. While carrying the process either during addition or subtraction any time the last digit is zero, which has to be ignored. Addition or subtraction check out the divisibility rule of some other numbers.

Basic concepts

Divisibility Test

Any integer a is said to be divisible by any integer b if there exist an integer c such that a = b c ($b\neq0$) in symbols we wrote $b \mid a$, read (b divided a) or a is divisible by b, if b does not divide a we wrote write $b \nmid a$.

Theorem (2-1)

If a, b, c, k and $\bar{k}$ are integers, then

(i) $a \mid a$ (ii) $1 \mid a$ (iii) $a \mid 0$

(iv) $a \mid b \wedge b \mid c \Rightarrow a \mid c$ (v) $a \mid b \Rightarrow a \mid b\, c$ (vi) $a \mid b \wedge b \mid a \Rightarrow b = \pm\, a$

Proof

(i) $a = a.\,1 \Rightarrow a \mid a$

(ii) $a = 1\,.\,a \Rightarrow 1 \mid a$

(iii) $0 = a\,.\,0 \Rightarrow a \mid 0$ ($a \neq 0$)

(iv) $a \mid b \Rightarrow b = a\,k$ (1)

$b \mid c \Rightarrow c = b\,\bar{k}$ (2)

from (1) and (2) we find

$c = a\,k\,\bar{k} \Rightarrow c = a\,(k\,\bar{k}) \Rightarrow a \mid c$

(v) $a \mid b \Rightarrow b = a\,k$ (3)

multiply both sides of (3) by c

$b\,c \Rightarrow a\,k\,c \Rightarrow b\,c = (k\,c) \Rightarrow a \mid b\,c$

(vi) $a \mid b \Rightarrow b = a\,k$ (4)

$b \mid a \Rightarrow a = b\,\bar{k}$ (5)

from (4) and (5) we find $b = b\,(\bar{k}\;k) \Rightarrow k\,\bar{k} = 1$ $k = \pm 1$

$\therefore\; b = \pm\, a$

Definition (2-2)

The integer p those divisors 1 and p is called prime number.

The set of prime numbers is $\{2, 3, 5, 7, \ldots\}$.

Definition (2-3)

If $a \geq 1$ is not prime number, then a is called composite.

Theorem (2-4)

If p is a prime number and $p \mid m\,n$ then $p \mid m$ or $p \mid n$.

Proof

If $p \nmid m$ then $(p, m) = 1 \Rightarrow p\,x + my = 1$ (6)

$x, y \in z$

multiply both sides of (6) by n

$n\,p\,x + n\,m\,y = n$ (7)

since $p \mid m\,n \Rightarrow m\,n = p\,k$ (8)

from (7) and (8) we find

$n\,p\,x + p\,k\,y = n \Rightarrow n = p\,(n\,x + k\,y) \Rightarrow p \mid n$

Similarly:

$p \nmid n$ then $(p, n) = 1$ and with the same steps we find $p \mid m$

Theorem (2-5)

Every $n > 1$ has at least one prime divisor.

Proof

Suppose d is a least positive integer divides n where $d > 1$, if d is a prime the condition satisfied. If d is not prime, let $d = d_1\,d_2$ where $1<d_1<d$ and since $d>1$ is divides n, which contradiction that d is a least prime divisor of n.

The concept of congruence

Definition (2-6)

Let n is a positive integer. Then as integer a congruent to an integer b modulo n if n | (a-b) in symbols, we write $a \equiv b \pmod{n}$.

Theorem (2-7)

$a \equiv b \pmod{n}$ if and only if $a = b + n\,k,\ k \in z$.

Proof:

$a \equiv b \pmod{n} \Rightarrow n \mid (a - b) \Rightarrow a - b = n\,k \Rightarrow a = b + n\,k$

conversely

if $a = b + n\,k \Rightarrow a - b = n\,k \Rightarrow n \mid (a - b)$ so that $a \equiv b \pmod{n}$.

Theorem (2-8)

If n is a positive integer a, b, c $\in \mathbf{Z}$

(i) $a \equiv a \pmod{n}$ (Reflexive property)

(ii) $a \equiv b \pmod{n},\ b \equiv a \pmod{n}$ (Symmetric property)

(iii) $a \equiv b \pmod{n},\ b \equiv c \pmod{n},\ b \equiv c \pmod{n}$ (Transitive property)

(The congruence rela1tion is an equivalence)

Proof

(i) $a \equiv a \pmod{n} \Rightarrow n \mid (a - a) \Rightarrow n \mid 0$

since $n \mid 0 \Rightarrow n \mid (a - a) \Rightarrow a \equiv a \pmod{n}$

(ii) $a \equiv b \pmod{n} \Rightarrow n \mid (a - b) \Rightarrow a - b = nk,\ k \in \mathbf{Z}$

$-(b - a)\ nk \Rightarrow b - a = n(-k)\ n \mid (a - b) \Rightarrow b \equiv a \pmod{n}$

(iii) $a \equiv b \pmod{n} \Rightarrow a - b \Rightarrow nk,\ k \in \mathbf{Z}$ (9)

$b \equiv c \pmod{n} \Rightarrow b - c \Rightarrow n\bar{k},\ k \in \mathbf{Z}$ (10)

add (9) + (10)

$a - c = n(k + \bar{k}) \Rightarrow n \mid (a - c) \Rightarrow a = c \pmod{n}$

congruence relation is (reflexive, symmetric, transitive). So that it's equivalence relation).

Theorem (2-9)

$a \equiv b \pmod{n}$ if and only if a and b have the same reminder when divided by n.

Proof

$a \equiv b \pmod{n} \Rightarrow a - b \Rightarrow a = b + nk,\ \ k \in \mathbf{Z}$

Form division algorithm $b = n\,q + r \quad 0 \le r < b$

$$a = b + nk = n\,q + r + nk = n\,(q + k) + r$$

therefore, a, b has the same reminder r when divided by n.

-conversely suppose both a and b have same remainder r when divided

From division algorithm

$$a = n\,q + r \tag{11}$$

$$b = n\,\bar{q} + r \tag{12}$$

subtract (11) – (12)

$$a - b = n\,q - n\,\bar{q} = n\,(q - \bar{q}) \Rightarrow n \mid (a - b)$$

$$\therefore \quad a \equiv b \pmod{n}.$$

Divisibility Test for (7, 13, 17)

Divisibility Test for 7

- Double the last digit of the number.
- Subtract it form the rest of the number.
- Repeat these two steps to find one- or two-digits' number and check it, to see if divisible by 7.

Example (3-1)

Check whether the number 24521 is divisible by 7.

Solution:

$$\begin{array}{r} 24521 \\ 2 \\ \hline 2450 \\ 0 \\ \hline 245 \\ 1\,0 \\ \hline 14 \end{array}$$

Since $7 \mid 14 \Rightarrow 7 \mid 24521$

Other solution

Multiply digits of number beginning form the last digit by 1, 3, 2, 6, 4 and 5 respectively and find the sum to check it, if is divisible by 7.

$1 \times 1 + 3 \times 2 + 2 \times 5 + 6 \times 4 + 4 \times 2 = 1 + 6 + 10 + 24 + 8 = 49$

Since $7 \mid 49 \Rightarrow 7 \mid 24521$

Divisibility Test for 13

- Multiple the last digit of the number by 4.
- Add it to the rest of the number.
- Repeat these two steps to find two digits' number and check it, to see whether the number is divisible by 13.

Example (3-2)

Prove that the number 957657 is divisible by 13.

Solution

$$\begin{array}{r} 957657 \\ 28 \\ \hline 95793 \\ 12 \\ \hline 9591 \\ 4 \\ \hline 963 \\ 12 \\ \hline 108 \\ 32 \\ \hline 42 \end{array}$$

$13 \mid 42 \Rightarrow 13 \mid 957657$

Example (3-3)

Check whether the number 395165 is divisible by 13.

Solution

395265
  20
39546
  24
3978
 32
 429
 36
 78

Since 13 | 78⇒13 | 395265

Divisibility Test for 17

- Multiple the last digit of the number by 5.
- Subtract it form the rest of the number.
- Repeat these two steps to find two digits' number and check it, to see whether the number is divisible by 17.

Example (3-4)

If the number 123352 is divisible by 17.

Solution

123352
  10
12325
  25
1207
 35
 85

Since 17 | 85⇒17 | 123352

Divisibility Test for (19, 23, 29)

Divisibility Test for 19

- Double the last digit of the number.
- Add it to the rest of the number.
- Repeat these two steps to find two digits' number and check it, to see if divisible by 7.

Example (4-1)

Prove that the number 955719 is divisible by 19 without doing division.

Solution:

$$
\begin{array}{r}
955719 \\
18 \\
\hline
95589 \\
18 \\
\hline
9576 \\
12 \\
\hline
969 \\
18 \\
\hline
114 \\
8 \\
\hline
19
\end{array}
$$

$19 \mid 19 \Rightarrow 19 \mid 955719$

i. **Divisibility Test for 23**

- Multiply the last digit of the number by 7.
- Add it to the rest of the number.
- Repeat these two steps to find two digits' number and check it, to see whether the number is divisible by 23.

Example (4-2)

Check whether the number 74635 is divisible by 23.

Solution:

74635
35
———
7498
56
———
805
35
———
115
35
———
46

Since 23 | 46 ⇒ 23 | 744635

Divisibility Test for 29

- Multiply the last digit of the number by 3.
- Add it to the rest of the number.
- Repeat these two steps to find two digits' number and check it, if divisible by 29.

Example (4-3)

If the number 93235 is divisible by 29.

Solution:

93235
15
———
9338
24
———
957
21
———
116
18
———
29

Since 29 | 29⇒ 29 | 93235

Example (4-4)

Check whether the number 62557 is divisible by 29.

Solution:

$$\begin{array}{r} 62557 \\ \underline{\quad 21} \\ 6276 \\ \underline{18} \\ 645 \\ \underline{15} \\ 79 \end{array}$$

Since $29 \nmid 79 \Rightarrow 29 \nmid 62557$

Conclusion

We reached the importance of numbers theory which it plays significant role in developing of applied science and technology.

In summary for any prime $p \neq 2, 5$ it is possible to determine a divisibility rule that is based on a trimming algorithm congruence relation.

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