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Residual Power Series Method and Adomian Decomposition Method for Solving Partial Differential Equations

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Abstract:

In This paper we present two methods, the first one is a powerful iterative method called Residual power series method introduced to obtain approximate solutions of nonlinear time-dependent generalized Fitzhugh-Nagumo equation with time-dependent coefficients and Sharma-Tasso-Olver equation subjected to certain initial conditions. In the second method we used Bernstein polynomials to modify the Adomian decomposition method which can be used to solve linear and nonlinear equations. Furthermore, the obtained results demonstrate reliability and activity of the proposed technique. Comparison with Other traditional technique

Keyward: *Residual power series method, nonlinear time-dependent generalized Fitzhugh-Nagumo equation, Sharma-Tasso-Olver equation, Bernstein polynomials, Adomian decomposition method.*

طريقة متسلسلة القوى المتبقية وطريقة التحلل الأدومي لحل المعادلات التفاضلية الجزئية

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المستخلص:

قدمنا طريقتين في هذه الورقة: الطريقة الاولي طريقه تكراريه قويه تسمي طريقةمتسلسلة القوي المتبقي للحصول علي الحلول التقريبية غير الخطية تابعة للزمن وبصوره عامه عرضنا معادلة (Filzhugh-Nayumo) التابعة للزمن و(Asharma-Tasso-Olver-Equation)لتؤكد الحالة الابتدائية. وفي الطريقة الثانيه:استخدمنا كثيرة حدود بيرنستن المعدلة لتعديل طريقة تركيب ادمون والتي يمكن استخدامهالحل المعادلات الخطية وغير الخطية. النتائج التي تم الحصول عليها تبرهن الموثوقية والفاعلية للطريقتين بالمقارنة مع التقنيات التقليدية الاخري.

كلمات مفتاحية:

طريقة متسلسلة القوى المتبقية، معادلة فيتزهوغ ناغومو المعممة غير الخطية المعتمدة على الزمن، معادلة شارما-تاسو-أولفر، كثيرات حدود برنشتاين، طريقة تحليل أدومايان.

Introduction:

We employ Residual Power Series Method (RPSM)to obtain the numerical solution for generalized Fitzhugh–Nagumo equation (FNE) with time-dependent coefficients [2] and Sharma–Tasso–Olver equation (STOE) [3].Nonlinear time-dependent generalized FNE [4]

$u_t + \cos(t)ux - \cos(t)u_{xx} + 2\cos(t)(u(1-u)(p-u)) = 0$

(1) $(x, t) \in [A,B]\times [0, T],\ 0 < p<1$

subjected to the initial condition

$$(2)\quad u(x,0) = \frac{\rho}{2} + \frac{\rho}{2}\sin\left(\frac{\rho x}{2}\right), \quad x \in [A,B]$$

Using specific solitary wave ansatz and the Tanh method (TanhM), new variety of soliton solutions are introduced and applied [Triki and Wazwaz, 2013; Bhrawy and Jacobi, 2013] the Jacobi–Gauss–Lobatto collocation method to solve the generalized FNE. In recent years, many physicists and mathematicians have paid much attention to the FNE on account of its importance in mathematical physics[Abbasbandy, 2008; Li and Gua, 2006]. The following nonlinear equation is obtained

$$(3)\quad u_t + \alpha\left(u^3\right) + \frac{3}{2}\left(u^3\right)_{xx} - \alpha u_{xxx} = 0$$

where α is a real parameter and u(x, t) is the unknown function depending on the variable t and x. Equation (3) be called STOE . The STOE applied in physical and sciences [Abu Arqub, 2013]. In addition,, fractional sub-equation method is used to construct exact solution of the nonlinear fractional STOE [Jafari *et al*., 2013].

2-Time-Dependent Generalized FNE

Consider generalized FNE with time-dependent coefficients(2) and (1).

The exact solution for equation (2) is

$$u(x,t) = \frac{\rho}{2} + \frac{\rho}{2}\tanh\left(\frac{\rho}{2}\left(x - (3-\rho)\sin(t)\right)\right),$$

We apply the RPSM to find out series solution for this equation subjected to given initial conditions by replacing its power series expansion with its truncated residual function. From this equation, a repetition formula for the calculation of coefficients is supplied ,while coefficients in power series expansion can be calculated repeatedly from the truncated residual function.[10]Suppose that the solution takes the expansion form

$$(4)\ u=\sum_{u=0}^{\infty} f_n(x)t^n,\ 0\le t<R,\ \ x\in I$$

Next, we let uk to denote kth, truncated series of u

$$(5)\ u_k=\sum_{u=0}^{k} f_n(x)t^n,\ 0\le t<R,\ \ x\in I$$

where $u_0=f_0(x)=u(x, 0)=f(x)$:Equation (5) can be written as

$$(6)\ u_k=f(x)+\sum_{u=0}^{k} f_n(x)t^n,\ 0\le t<R,\ \ x\in I, k=\overline{1,\infty}$$

First, to find the value of coefficients $f_n(x)$,n=1, 2, 3, …, k in series expansion of equation(5), we define residual function Res, for equation (1), as

$$\operatorname{Re}s=u_t+\cos(t)u_x-\cos(t)u_{xx}+2\cos(u(1-u)(\rho-u))$$

and the kth residual function, Res_k , as follows

$$(7)\ \begin{aligned}\operatorname{Re}s_k=(u_k)_t+\cos(t)(u_k)_x-\cos(t)(u_k)_{xx}\\ +2\cos(t)(u_k(1-u_k)(\rho-u_k))\end{aligned}$$

k=1, 2, 3, …

it is clear that Res=0 and $\lim_{k\to k}\operatorname{Re}s_k\ \operatorname{Re}s$

for each $x\in I$ and t = 0.

Then, $(\partial^r \mathrm{Res}=\partial t^r)=0$ when t=0 for each r= $\overline{0,k}$. To determine $f_1(x)$, we write k=1 in equation (7)

$$(8)\ \begin{aligned}\operatorname{Re}s_k=(u_k)_t+\cos(t)(u_k)_x-\cos(t)(u_k)_{xx}\\ +2\cos(t)(u_1(1-u_1)(\rho-u_1))\end{aligned}$$

where

$u_1=f(x)+tf_1(x)$

for

$$u_0=f_0(x)=f(x)=u(x,t)=\frac{\rho}{2}+\frac{\rho}{2}\tanh\left(\frac{\rho x}{2}\right)$$

From equation (7), we deduce that $\mathrm{Res}_1=0$ (t=0)and thus

$$(9)\ f_1(x)=\frac{1}{4}(-3+p)\rho^2\,\operatorname{sec}h^2\left(\frac{\rho x}{2}\right)$$

Therefore, the 1st residual power series (RPS)approximate solutions are

$$(10)\ u_1 = \frac{\rho}{2} + \frac{\rho}{2}\tanh\left(\frac{\rho x}{2}\right) + \frac{1}{4}(-3+p)\rho^2 \operatorname{sec h}^2\left(\frac{\rho x}{2}\right)t$$

Similarly, to find out the form of the second unknown coefficient, $f_2(x)$, we write

$u_2=f(x)+tf_1(x)+t^2f_2(x)$

in Res_2.

$(\partial Res_2=\partial t)=0$ $(t=0)$ and thus

$$(11)\ f_2(x) = -\frac{1}{8}(-3+p)^2\rho^3 \operatorname{sec h}^2\left(\frac{\rho x}{2}\right)\tanh\left(\frac{\rho x}{2}\right)$$

Therefore, the 2nd RPS approximate solutions are

$$(12)\ \begin{aligned} u_2 &= \frac{\rho}{2} + \frac{\rho}{2}\tanh\left(\frac{\rho x}{2}\right) + \frac{1}{4}(-3+p)\rho^2 \operatorname{sec h}^2\left(\frac{\rho x}{2}\right)t \\ &-\frac{1}{8}(-3+p)^2\rho^3 \operatorname{sec h}^2\left(\frac{\rho x}{2}\right)\tanh\left(\frac{\rho x}{2}\right)t^2. \end{aligned}$$

Similarly, we write

$U_3=f(x)+tf_1(x)+t^2f_2(x)+t^3f_3(x)$

in Res_3.

$(\partial^2 Res_3=\partial t^2)=0$ $(t=0)$ and thus

$$(13)\ \begin{aligned} f_3(x) &= \frac{1}{48}\rho^2\big(-3+p^2-54\rho^3+18\rho^4-2\rho^5 \\ &+3\cosh(\rho x)-\rho\cosh(\rho x)-27\rho^2\cosh(\rho x) \\ &+27\rho^3\cosh(\rho x)-9\rho^4\cosh(\rho x) \\ &+\rho^5 s\cosh(\rho x)\big)\operatorname{sec h}^4\left(\frac{\rho x}{2}\right) \end{aligned}$$

Therefore, the 3rd RPS approximate solutions are

$$(14)\ \begin{aligned} u_3 &= \frac{\rho}{2} + \frac{\rho}{2}\tanh\left(\frac{\rho x}{2}\right) + \frac{1}{4}(-3+p)\rho^2 \operatorname{sec h}^2\left(\frac{\rho x}{2}\right)t \\ &-\frac{1}{8}(-3+p)^2\rho^3 \operatorname{sec h}^2\left(\frac{\rho x}{2}\right)\tanh\left(\frac{\rho x}{2}\right)t^2 \\ &+\frac{1}{48}\rho^2\big(-3+p^2-54\rho^3+18\rho^4-2\rho^5 \\ &+3\cosh(\rho x)-\rho\cosh(\rho x)-27\rho^2\cosh(\rho x) \\ &+27\rho^3\cosh(\rho x)-9\rho^4\cosh(\rho x) \\ &-9\rho^5 s\cosh(\rho x)\big)\operatorname{sec h}^4\left(\frac{\rho x}{2}\right) \end{aligned}$$

Consider equation (3) with the initial condition [Jawad *et al*., 2010].

$$u(x,t)=\frac{1}{1+e^{-x}}$$

The exact solution for equation (3) is

$$u(x,t)=\frac{1}{1+e^{-(x-\alpha t)}}$$

We apply the RPSM to find out series solution for this equation.

$$(15)\quad u=\sum_{n=0}^{\infty} f_n(x)t^n,\quad 0\le t<R,\quad x\in I$$

where u_k is the truncated series of u

$$(16)\quad u_k=\sum_{n=0}^{\infty} f_n(x)t^n,\quad 0\le t<R,\quad x\in I$$

where $u_0=f_0(x)=u(x, 0)=f(x)$.

To find the value of coefficients $f_n(x)$,n=1, 2, 3, …, k in series expansion of equation (5),we define residual function Res, for equation (3), as

$$Res=u_t+\alpha\left(u^3\right)_x+\frac{3}{2}\alpha\left(u^2\right)_{xx}+\alpha u_{xxx}$$

and the kth residual function, Res_k , as follows

$$(17)\quad Res_k=\left(u_k\right)_t+\alpha\left(u^3{}_k\right)_x+\frac{3}{2}\alpha\left(u_k{}^2\right)_{xx}+\alpha(u)_{xxx_k}$$

k = 1,2,3, …

To determine $f_1(x)$, we write k=1 in equation (17)

$$(18)\quad Res_1=\left(u_1\right)_t+\alpha\left(u^3{}_1\right)_x+\frac{3}{2}\alpha\left(u_1{}^2\right)_{xx}+\alpha\left(u_1\right)_{xxx}$$

where

$u_1=f(x)+tf_1(x)$

for

$$u_0=f_0(x)=f(x)=u(x,0)=\frac{1}{1+e^{x}}$$

From equation (18), we deduce that $Res_1=0(t=0)$ and thus

$$(19)\quad u_0=\frac{1}{1+e^{-x}}-\frac{e^{x}\alpha}{\left(1+e^{x}\right)^2}t$$

Similarly, to find out the form of the second unknown coefficient, $f_2(x)$, we write

$u_2=f(x)+tf_1(x)+t^2f_2(x)$

in Res_2.

$(\partial Res_2=\partial t)=0$ (t=0) and thus

$$(20)\quad f_2(x)=\frac{e^x(-1+e^x)\alpha^2}{2(1+e^x)^3}$$

Therefore, the 2nd RPS approximate solutions are

$$u_2=\frac{1}{1+e^{-x}}-\frac{e^x\alpha}{(1+e^x)^2}t-\frac{e^x(-1+e^x)\alpha^2}{2(1+e^x)^3}t^2$$

Similarly, we write

$u_3=f(x)+tf_1(x)+t^2f_2(x)+t^3f_3(x)$

in Res_2.

$(\partial^2 Res_2=\partial t^2)=0$ (t=0) and thus

$$(21)\quad f_3(x)=-\frac{e^x(-1-4e^x+e^{2x})\alpha^3}{6(1+e^x)^4}$$

Therefore, the 3rd RPS approximate solutions are

$$(22)\quad u_3=\frac{1}{1+e^{-x}}-\frac{e^x\alpha}{(1+e^x)^2}t-\frac{e^x(-1+e^x)\alpha^2}{2(1+e^x)^3}t^2-\frac{e^x(-1-4e^x+e^{2x})\alpha^3}{6(1+e^x)^4}$$

and

$$(23)\quad f_4(x)=-\frac{e^x(-1-11e^x-11e^{2x}+e^{3x})\alpha^4}{24(1+e^x)^5}$$

3-Adomian Decomposition Method with Modified Bernstein Polynomials**:**

Definition(1)

. The Bernstein basis polynomials of degree m over the interval [0, 1] are defined by

$$(24)\quad B_{i,m}(x)=\binom{m}{i}x^i(1-x)^{m-1}$$

where the binomial coefficient is

$$(25)\binom{m}{i} = \frac{m!}{!(m-i)!}$$

For example, when m=5, then the Bernstein terms are

$B_{2,5}(x) = 10x^2(1-x)^3$

(26)$2B^{3,5}(x) = B_{0,5}(x) = (1-x)^5$

$B_{1,5}(x) = 5x(1-x)^4$

$10x^3(1-x)^2$

$B^{4,5}(x) = 5x^4(1-x)$

$B^{5,5}(x) = x^5$

Definition (2) Bernstein polynomials

A linear combination of Bernstein basis polynomials

$$(27)\, B_m(x) = \sum_{i=0}^{m} B_{m,i}(x)\beta_i$$

is called the Bernstein polynomials of degree m, where $\beta_i$arethe Bernstein coefficients.[Ahmed and Ekhlass, 2018]

Definition (3)

Let f be a real valued function defined and bounded on [0, 1]; let $B_m(f)$ be the polynomial on [0, 1],defined by

$$(28)\, B_m(f) = \binom{m}{i} x^i (1-x)^{m-1} f\left(\frac{i}{m}\right)$$

where $B_m(f)$ is the m-th Bernstein polynomials for f(x).
For each function f : [0,1] →R, we have

$$(29)\, \lim_{m\to\infty} B_m^f(x) = f(x)$$

Example If f (x) = e^x, x ∈ [0, 1] then the Bernstein expanded for the function f (x) when m=5 is

$$B_m(f) = f(0)(1-x)^5 + f\left(\frac{1}{5}\right)5x(1-x)^4 + f\left(\frac{2}{f}\right)10x^2(1-x)^3$$
$$+ f\left(\frac{4}{5}\right)5x^4(1-x)^4 + f(x)x^5$$

$$(30)\, B_m(f) = e^0(1-x)^3 + 5e^{1/5}x(1-x)^3 + 10e^{2/5}x^2(1-x)^2$$
$$+10e^{3/5}x^3(1-x) + 5e^{4/5}x^4(1-x) + e^1x^5$$

If the 2k-th order derivative $f^{2k}(x)$ is bounded in the interval (0,1) then for each $x \in [0,1]$.[Lorentz, 1986].

$$(31)\quad B_m^f(x) = f(x) + \sum_{a=0}^{2k-1} \frac{f^{(a)}(x)}{a!} T_{m,a}(x) + O\left(\frac{1}{m^k}\right)$$

where

$$(32)\quad T_{m,a}(f) = \sum_k (k - mx)^a \binom{m}{k} x^k (1-x)^{m-k}$$

Remark. Notice that $T_{m,a}(x)$ is the a-th central moment of a random variable with a binomial appropriation with parameters m and x. Clearly, $T_{m,0}$,0 = 1, $T_{m,1} = 0$. It is well known that the sequence $\{T_{m,a}(x)\}$ satisfies the following recurrence.

$$(33)\quad T_{m,a+1}(x) = x(1-x)(T_{m,a}(x) + maT_{m,a-1}(x))$$

If we apply (30) to k = 1; 2; 3, then we obtain

$$B_m^f(x) = f(x) + O\left(\frac{1}{m}\right)$$

$$B_m^f(x) = f(x) + \frac{x(1-x)f''(x)}{2m} + O\left(\frac{1}{m^2}\right)$$

$$B_m^f(x) = f(x) + \frac{x(1-x)f''(x)}{2m} + \frac{x(1-x)(4(1-2x)f^{(3)}(x) + 3x(1-2x)f^{(4)}(x))f''(x)}{24m^2} + O\left(\frac{1}{m^3}\right) \quad (34)$$

Let us consider the following equation:

Lu + Nu + Ru = g (x) (35)

where L is an invertible linear term, N represents the nonlinear term, and R is the remaining linear part; from (34)we have

Lu = g (x)− Nu−Ru. (36)

Now, applying the inverse factor L^{-1} to both sides of (36) then by the initial conditions we find

(37) $u = f(x) - L^{-1}Nu - L^{-1}Ru$,

where $L^{-1} - \int_0^x (.)dx$ f(x) are the terms having from integrating the rest of the term g (x) and from utilizing the given initial or boundary conditions. The ADM assumes that N(u) (nonlinear term) can be decomposed by an infinite series of polynomials which is expressed in form

(38) $$N(u)=\sum_{n=0}^{\infty}A_n(u_0,u_1,\cdots,u_n)$$

where A_n are the Adomian's polynomials defined as

(39) $$A_n=\frac{1}{n!}\frac{d^n}{d\lambda^2}\left[N\left(\sum_{i=0}^{\infty}\lambda^i u_i\right)\right]_{\lambda=0},\quad n=0,1,2,\cdots$$

(40) $$g(x)=\sum_{i=0}^{m}a_i B_i(x)$$

where $B_i(x)$ is the Bernstein polynomials .Now, using (37) and (40) we have

(41) $$\begin{aligned}u_0&=L^{-1}(a_0B_0(x)+a_1B_1(x)+a_2B_2(x)+\cdots+a_mB_m(x))+\theta(x),\\u_1&=-L^{-1}(Ru_0)-L^{-1}(Nu_0)\\u_2&=-L^{-1}(Ru_1)-L^{-1}(Nu_1)\\&\vdots\end{aligned}$$

and so on[13]

we improve the function g(x) using modified Bernstein series

(42) $$g(x)=\sum_{i=0}^{m}\binom{m}{i}x^i(1-x)^{m-i}f\left(\frac{i}{m}\right)-\sum_{a=0}^{2k-1}\frac{f^{(a)}(x)}{a!}T_{m,a}(x)$$

And we can approach the derivatives using the Bernstein polynomials

(43) $$\frac{d}{dx}B_{i,m}(x)=m(B_{i-1,m}(x)-B_{i,m-1}(x)),$$

Then (4-61) becomes

(44) $$g(x)=\sum_{i=0}^{m}\binom{m}{i}x^i(1-x)^{m-i}f\left(\frac{i}{m}\right)-\sum_{a=1}^{2k-1}\frac{\left(d^{(a)}dx^{(a)}\right)B_{i,m}(x)}{a!}T_{m,a}(x)$$

Now, using (41) and (44) we have

(45) $$\begin{aligned}u_0&=L^{-1}(B_{i,m}(x))+\Theta(x),\\u_1&=L^{-1}(Ru_0)+L^{-1}(Nu_0),\\u_2&=L^{-1}(Ru_1)+L^{-1}(Nu_1),\\&\vdots\end{aligned}$$

The above equation is governing equation of ADM using modified Bernstein polynomials. The obtained approximate solution, Wy(x) = Yj=$_0$^{u}i, by (45) has a comparison with the classic approximation solution and the correct solution[14-15].

Example (1) [Ahmed and Ekhlass, 2018**]** Consider the ordinary equation

$$(46)\ \frac{d^2y}{dt^2} + t\frac{dy}{dt} + t^2y^3 = \left(2+6t^2\right)e^{t^2} + t^2e^{3t^2},\ \ y(0)=1,$$

$$\frac{dy(0)}{dt} = 0$$

$\frac{dy}{dx}$ with the exact solution (t) = e^{t^2} . Using (35)we have

Ly + Ny + Ry = g (x) (47)

where L = d^2/dt^2, Ry = t(d/dt),Nyt^2y^3, and d(x) = (2 +6t^2)e^{t^2} + $t^2e^{3t^2}$.

The Adomian polynomials for representing the nonlinear term Ny are

$$A_0 = t^2y_0^3,$$
$$(48)\ A_1 = t^2\left(3y_0^3y_1\right),$$
$$A_2 = t^2\left(3y_0^3y_2 + 3y_0^3y_1^2\right),$$
$$\vdots$$

Now $L^{-1} = \int_0^t\int_0^t(.)dtdt$; then using (28) the classical Bernstein polynomials of g(t) when

v=m=6

are

$$(49)\ g_b(x) = 2 + 1.547324t + 9.290164t^2 + 7.83289t^3 + 9.751887t^4 + 7.659668t^5 + 3.749864t^6$$

And modified Bernstein polynomials (4-63) of g(x) with k=2 are

$$(50)\ g_{mb}(x) = 2 - 0.001037t + 6.922082t^2 + 1.997441t^3 + 6.737662t^4 + 11.051121t^5 + 13.124523t^6$$

By (45), we have

$$y_0 = L^{-1}(g_{mb}(t)) + y(0) + \frac{dy}{dx}(0)t = 1 + t^2 - 0.000173t^3 + 0.57684t^4 + 0.099872t^5 + 0.224589t^6 + 0.263122t^7 + 0.234367t^8,$$

$$(51)\ y_1 = -L^{-1}\left(t\frac{d}{dt}y_0\right) - L^{-1}(A_0) = -0.25t^2 + 0.000026t^5 - 0.176912t^6 - 0.011877t^7 + \cdots,$$

$$y_2 = -L^{-1}\left(t\frac{d}{dt}y_1\right) - L^{-1}(A_1) = 0.033333t^6$$
$$-0.000003t^7 + 0.032348t^8 - 0.011536t^7 + \cdots,$$
$$\vdots$$

And we obtain

$$(52)\quad y_{m,b}(t) = \sum_{i=0}^{6} y_i = 1 + t^2 - 0.000173t^3 + 0.32684t^4 + \cdots.$$

Conclusion

The RPSM is applied successfully for solving the generalized FNE with time-dependent coefficients and STOE for certain initial conditions. it is concluded that the RPSM becomes powerful and efficient in finding numerical solutions for a wide class of nonlinear differential equations, we show that utilizing modified *Bernstein Polynomials Through to modify performsnce*of the Adomian decomposition technique. it can be used specifically for all sort of differential and integral equations.

Results:

(1) The main advantage of [RPSM] is the simplicity in calculating the coefficients of terms of the series solution using only the differential operators.

(2) The fundamental stand point of the Adomian decomposition technique is that it can be used specifically for all sort of differential and integral equations.

(3) We utilize modified Bernstein extensions of the nonlinear term to get more exact outcomes.

References:

Abbasbandy S. (2008). Soliton solutions for the Fitzhugh- Nagumo equation with the homotopy analysis method. Appl Math Model; 32: 2706-2714.

Abu Arqub O. (2013). Series solution of fuzzy differential equations under strongly generalized differentiability. J AdvRes Appl Math; 5: 1-52.

Ahmed Faroog Qasim and EKhlass S.AL-Rawi (2018). Adomian Decomposition Method with Modified Bernstein polynomials for Solving Ordinary and partial Differential Equations. Mosul, Irag.

Bhrawy, A.H.; A Jacobi-Gauss-Lobatto (2013). collocation method for solving generalized Fitzhugh-Nagumo equation with time-dependent coefficients. Appl Math Comput; 222: 255-264.

El-Ajou A, Abu Arqub O, Al Zhour Z, et al. New results on fractional power series: theories and applications. Entropy 2013; 15: 5305-5323.

He JH. (1999). Homotopy perturbation technique. Comput Method Appl M 1999; 178: 257-262.

Jafari H.; Tajadodi, H.; Baleanu, D., *et al*. (2013). Fractional sub-eq41 method for the fractional generalized reaction Duffing model and nonlinear fractional Sharma- Tasso-Olver equation. Cent Eur J Phys; 11: 482-1486.a

Jawad, AJM; Petkovic, MD and Biswas, A. (2010). Modified simple equation method for nonlinear evolution equations. Appl Math Comput; 217: 869-877.

Li H and Guo Y. (2006). New exact solutions to the Fitzhugh- Nagumo equation.Appl Math Comput; 180: 524-528.

Liu, Y. (2009). "Adwoman decomposition method with orthogonal polynomials: Legendre polynomials," Mathematical and Computer Modelling, vol. 49, no. 5-6, pp. 1268-1273.

Lorentz, G.G. (1986). BernsteinPolynomials,Chelsea publishing Series.

Mustafa Inc, Zeliha Skorpinar, Maysauu, Momd ALqurashi and Durnitru Baleanu (2016). Anew method for approximate solutions of some nonlinear equations:Residual power series method-Turkey.

Rani, D. and Mishra, V. (2017). "Approximate solution of boundary value problem with bernstein polynomial laplace decomposition method," International Journal of Pure and Applied Mathematics, vol. 114, no. 4, pp. 823-833.

Triki, H. and Wazwaz, A-M (2013). On soliton solutions for the Fitzhugh-Nagumo equation with time-dependent coefficients. Appl Math Model; 37: 3821-3828.

Yu, B.; Jiang, X.; Xu, H. (2015). A novel compact numerical method for solving the two dimensional non-linear fractionalreactionsubdiffusionequation. Numer. Algorithms, 68, 923–950.